



Wavelet leader multifractal analysis of heart rate variability in atrial fibrillation

Kais Gadhomi, PhD^{a,*}, Duc Do, MD, PhD^e, Fabio Badilini, PhD^b, Michele M. Pelter, RN, PhD^a, Xiao Hu, PhD^{a,c,d,f}

^a Department of Physiological Nursing, University of California, San Francisco, CA, USA

^b Center for Physiologic Research, University of California, San Francisco, CA, USA

^c Institute for Computational Health Sciences, University of California, San Francisco, CA, USA

^d Department of Neurological Surgery, University of California, San Francisco, CA, USA

^e David Geffen School of Medicine, University of California, Los Angeles, CA, USA

^f Department of Neurosurgery, University of California, Los Angeles, CA, USA

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ABSTRACT

Background: Accurate and timely detection of atrial fibrillation (AF) episodes is important in primarily and secondary prevention of ischemic stroke and heart-related problems. In this work, heart rate regularity of ECG inter-beat intervals was investigated in episodes of AF and other rhythms using a wavelet leader based multifractal analysis. Our aim was to improve the detectability of AF episodes.

Methods: Inter-beat intervals from 25 ECG recordings available in the MIT-BIH atrial fibrillation database were analysed. Four types of annotated rhythms (atrial fibrillation, atrial flutter, AV junctional rhythm, and other rhythms) were available. A wavelet leader based multifractal analysis was applied to 5 min non-overlapping windows of each recording to estimate the multifractal spectrum in each window. The width of the multifractal spectrum was analysed for its discrimination power between rhythm episodes.

Results: In 10 of 25 recordings, the width of multifractal spectrum was significantly lower in episodes of AF than in other rhythms indicating increased regularity during AF. High classification accuracy (95%) of AF episodes was achieved using a combination of features derived from the multifractal analysis and statistical central moment features.

Conclusions: An increase in the regularity of inter-beat intervals was observed during AF episodes by means of multifractal analysis. Multifractal features may be used to improve AF detection accuracy.

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Introduction

Atrial fibrillation (AF) is a leading risk factor for ischemic stroke [1]. Paroxysmal episodes of AF occur in 25 to 62% of cases [2], are frequently asymptomatic, and their detection can be elusive without continuous screening and accurate detection tools. A plethora of algorithms have been developed to detect AF from ECG. Many of these algorithms are based on statistical analysis of heart rate variability using temporal, spectral, geometrical or entropy measures [3–7].

In this work, we explore the higher order statistical properties of AF rhythm using multifractal analysis, a tool that has largely been applied in HRV analysis [8,9]. Wavelet based fractal analysis has been applied to ECG inter-beat interval for the classification of sinus and AF rhythms [4]. The authors used discrete wavelet analysis and fractal analysis to estimate respectively a variability index and the Hurst exponent and

reportedly succeeded to reach specificity and sensitivity levels exceeding 92%. Zhou et al. [10] demonstrated that ECG inter-beat intervals show multifractal properties using multifractal detrended fluctuation analysis and that generalized Hurst exponents can distinguish between normal and AF rhythms.

It was shown that robust and practically efficient formulation of multifractal analysis should be based on wavelet leaders, a derivation of wavelet coefficients of the wavelet transform [11]. Wavelet leader multifractal analysis was used with HRV analysis to demonstrate changes in the multifractal properties of ECG before myocardial ischemia [12] and to discriminate between survival and non-survival patients with congestive heart failure [13].

Using wavelet based multifractal analysis, we analyse the temporal fluctuation of the local regularity of ECG inter-beat interval in episodes of AF and other rhythms and evaluate the practicality of multifractal features in the discrimination of AF from other rhythms. We hypothesise that atrial fibrillation rhythm, although irregular, is not a random process, and that preferential conduction patterns leading to various degrees of short-term predictability may lead to identifiable regularity in

* Corresponding author at: Department of Physiological Nursing, University of California, San Francisco (UCSF), 2 Koret Way, San Francisco, CA 94143-0610, USA.

E-mail address: kais.gadhomi@ucsf.edu (K. Gadhomi).

the AF rhythm [14]. Ultimately, we propose new measures in a goal to help reaching high and robust detection accuracy of AF from the sequence of inter-beat intervals.

Material and methods

Data

Inter-beat (RR) intervals of 25 ECG recordings from human subjects available in the MIT-BIH atrial fibrillation database were investigated [7,15]. QRS peaks and annotations of 4 types of rhythms (atrial fibrillation (AF), atrial flutter (AFL), AV junctional rhythm (J), and other rhythms (N)) in each ECG recording were provided in the database and were used in the analysis. Of the 25 available ECG recordings, 2 contained only one rhythm type (AF), 12 contained 2 types of rhythms (AF and N), 10 contained 3 types of rhythms (AF, N, and AFL or J), and 1 contained 4 rhythms (AF, AFL, J, and N). ECG recordings were 10 h-long each and were sampled at 250 Hz with 12-bit resolution over a range of ± 10 mV.

To ensure statistical conclusion validity, only recordings in which AF was present with at least one other rhythm (AFL, J, or N) and for which the episodes were long enough (defined as episodes generating 10 data points or more using a sliding window analysis) to carry a statistical comparison were included in the study. For the analysis window length chosen in the study (5 min; see Methods for more details), 15 of 25 recordings were excluded because of this selection criterion.

Wavelet leader multifractal analysis

To measure the regularity of inter-beat intervals, we analysed the fluctuation of the local regularity across time using multifractal analysis. In multifractal analysis, the local regularity of data is measured by the scaling exponent h (also called Hölder exponent and Hurst exponent). Strong and sharp singularities in data are characterized by values of h close to zero while smooth singularities will lead to large values of h . The so-called multifractal spectrum $D(h)$ (Fig. 1) indicates the

distribution of the scaling exponents in the data and measures the variability of the local regularity in time. If w_D designates the width of the multifractal spectrum $D(h)$ (i.e. $w_D = h_{max} - h_{min}$), small values of w_D indicate *monofractal* data, in which the local regularity does not significantly vary in time, while large values of w_D indicate *multifractal* data, which exhibit variations of the local regularity over time.

The direct numerical computation of the multifractal spectrum $D(h)$ from the scaling exponents h is limited by the finite resolution and the sampling of signal [16]. The multifractal formalism provides an alternative approach to computing the multifractal spectrum based on the calculation of the structure functions [17].

In this study the wavelet leader multifractal formalism was used. The multifractal spectrum is estimated using structure functions determined for the linearly-spaced moments from -5 to 5 . The structure functions are computed based on the wavelet leaders which are obtained using biorthogonal spline wavelet filter (we have used the biorthogonal filter with one vanishing moment in the synthesis wavelet and five vanishing moments in the analysis wavelet).

Wavelet leader multifractal analysis (WLMA) is a multifractal formalism that uses the so-called *wavelet leaders* – a subset of discrete wavelet transform coefficients derived from the localized suprema of these coefficients – to estimate scaling exponents and the corresponding multifractal features (namely the log-cumulants $c_p, p = 1, 2, 3, \dots, N$. cf. [18,19] for detailed description of WLMA).

We extracted the scaling exponents and log-cumulants and calculated the width of multifractal spectrum w_D , in non-overlapping 5-min windows across each ECG recording. The length of the window was chosen based on the minimum number of data samples required to get a robust multifractal spectrum estimation. This number depends primarily on the type of mother wavelet used. In this work, we used the biorthogonal mother wavelet, which is often more adapted to derive the properties of random processes [18] and for which at least 248 data samples are required. This corresponds to a minimum window length of ~ 4 min for an average inter-beat interval of 1 s. To account for faster AF rhythms, we chose a slightly longer window.

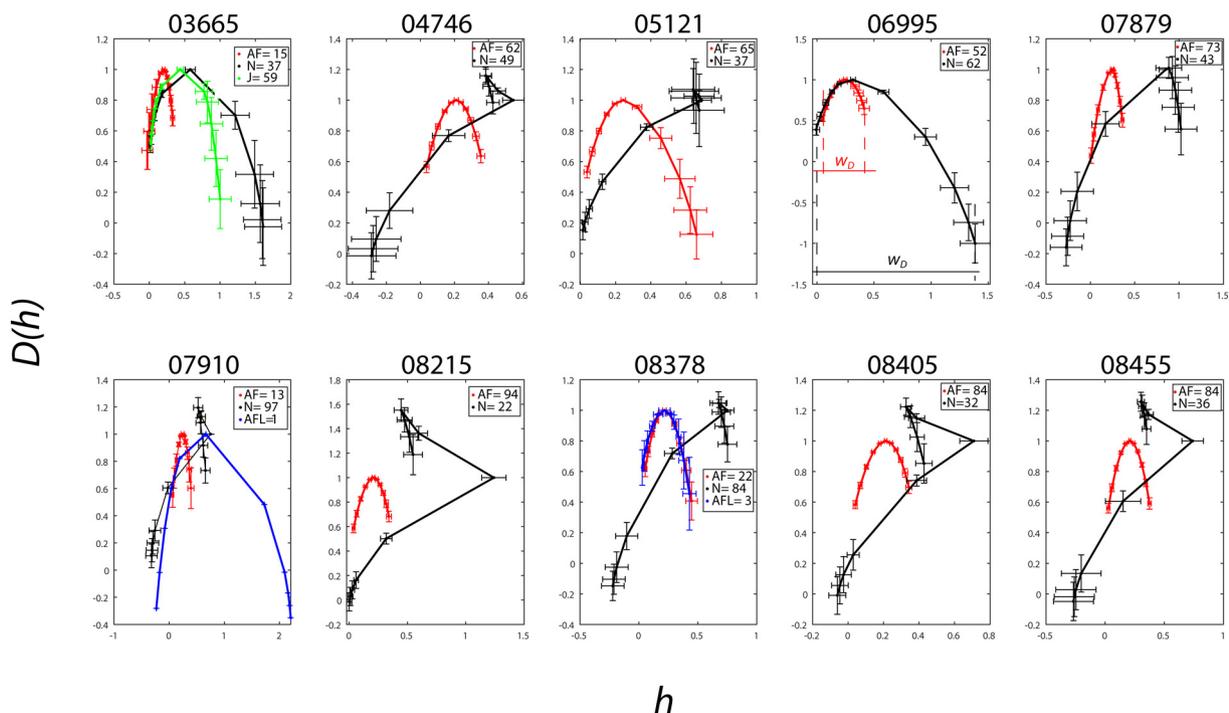


Fig. 1. Average multifractal spectra of available rhythm episodes (AF, AFL, J, and N) in 10 recordings. Panel 06995 illustrates the multifractal spectrum width w_D in AF and N episodes. Panel legends indicate the number of windows (i.e. number of spectra) in each rhythm episode.

Statistical analysis

Distributions of multifractal spectrum width values w_D calculated in the 5-min non-overlapping windows were compared between the rhythm episodes (intra-recording) of each recording and across all recordings (inter-recordings). To avoid uncertainty around the classification of the rhythm in each window, only episodes in which the entire 5-min window was AF or other rhythm were analysed. The temporal variation of w_D across each recording was analysed for consistency with corresponding episodes.

Rhythm classification

To evaluate the usefulness of the multifractal spectrum width as a feature for classifying the underlying rhythm of inter-beat intervals, we evaluated the performance of a series of machine learning classifiers based on support vector machines, ensemble learners, decision trees, nearest neighbor, and discriminant analysis, using the set of multifractal features ($w_D, c_p; p = 1,2,3$) as features. In total 22 different models were trained. Additionally, and for comparison with conventional statistical descriptors, we trained and evaluated these models using central moments (variance, skewness, and kurtosis – calculated from the same non-overlapping windows) and using the combination of multifractal features with central moments. Aggregate feature data extracted from all recordings were split into non-stratified random partitions. 75% of the data were used for training and 25% of the data were held for ultimate testing. A10-fold cross validation was performed on the training data set to avoid over-fitting: In each training fold $k, k = 1:10$, all classifiers were trained using a random partition P_k , made of 80% of the training data, and their performances were evaluated on the remaining 20% of the training data. The classifier with best average classification accuracy across all 10 training folds was selected. Its performance was then ultimately tested using the testing dataset.

Results

Comparison of the average multifractal spectrum width

Intra-recording comparison

Fig. 1 shows the average spectral for 10 recordings which fulfilled the above condition. The width of the average multifractal spectrum was remarkably smaller ($p < 0.01$, one-way ANOVA) for AF episodes than for AFL, J, and N episodes (see Supplemental Fig. 1).

A remarkable reduction in the value of w_D during AF episodes was observed (Fig. 2).

Inter-recording comparison

AF episodes from all recordings showed smaller w_D values compared with other episodes ($p < 0.01$, one-way ANOVA). Pairwise comparison between episodes showed statistically significant difference between AF and each of the other three episodes. Difference between values of w_D in AFL, J and N were not statistically significant (see Supplemental Fig. 2).

Classification

With AF rhythm being the positive class and AFL, J and N rhythms collectively the negative class, training accuracy was highest with bagged trees ensemble classifiers. This result is independent from the choice of feature vectors: multifractal features, central moments or both sets. Classification accuracy was higher using all multifractal features than using any subset of these features. Central moments were better predictors of AF than multifractal features (better training and testing accuracy). The combination of both set of features led to incremental increase of the overall classification performance evaluated on the testing set, with 94.9% accuracy and an AUC of 99.0%. Table 1 and Fig. 3 show respectively the testing performance and ROC curves of the bagged tree classifier for each set of features.

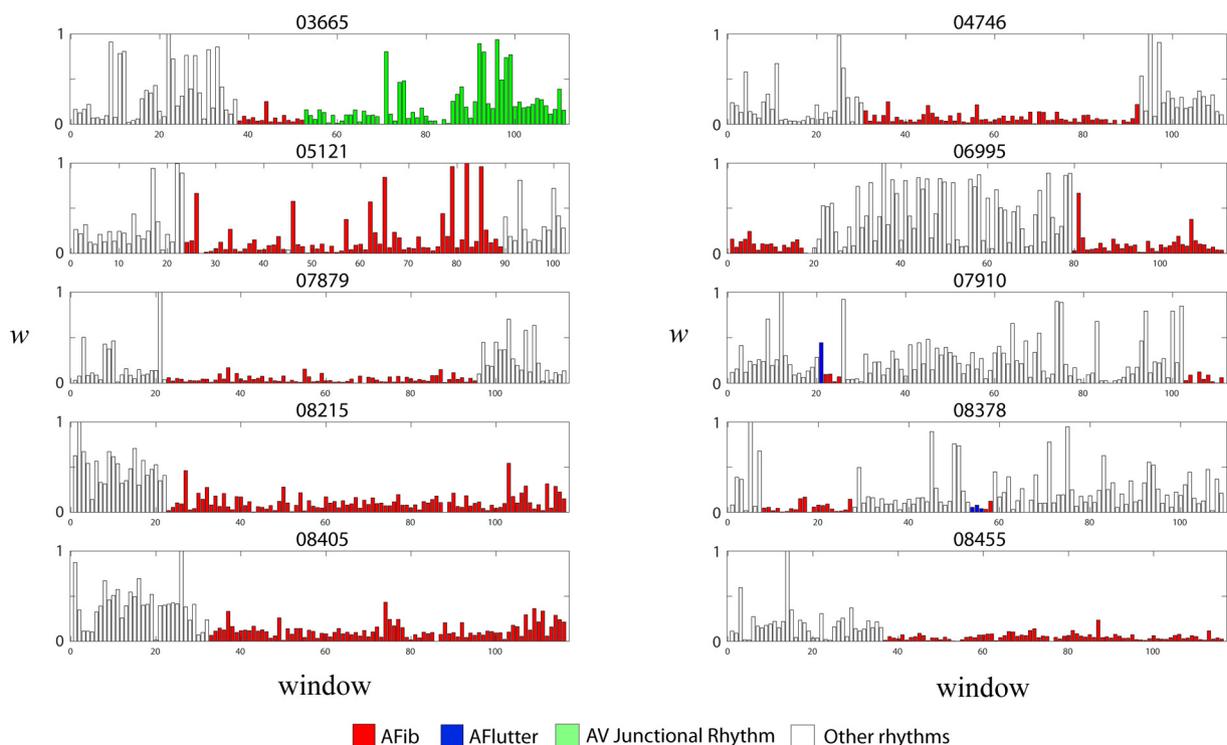


Fig. 2. Temporal variation of the normalized multifractal spectral width. Each value represents one 5-min window.

Table 1
Performance of bagged tree ensemble classifiers.

	Multifractal features	Central moments	Combined sets
Accuracy	80.4	93.1	94.9
Sensitivity	83.3	97	97.5
Specificity	78.2	91.2	95
F1 score	84.5	95.8	97.2
Prevalence	61	61	61
AUC	89.2	97.9	99.2
PPV	85.6	94.5	96.8
NPV	75	95.2	96.1

AUC: Area Under the Curve. PPV: Positive Predictive Value. NPV: Negative Predictive Value. AF is the positive class. AFL, J, and N are collectively the negative class. Total observations (w_D values) in dataset = 2676.

Discussion

Detection of atrial fibrillation from ECG often relies on the irregularity of inter-beat interval as a characterizing feature. This irregularity is manifested by higher variance in the inter-beat interval during episodes of AF compared with episodes of normal/other sinus rhythms. Many studies proposed AF detectors based on the variance or other low-order statistical measures of the inter-beat interval and reported moderate to high detection accuracies [6,20–22]. Other studies used complexity measures (e.g. entropy measures) to characterize and detect AF episodes from the inter-beat tachogram [23] or raw ECG [24].

In this work, our aim was to analyse the scaling behavior in the heart rate variability during AF episodes. An appropriate tool for the analysis of such scaling behavior is the wavelet based multifractal analysis, which closely reproduces and characterizes the scaling properties that exist in data [18]. Recently, the introduction of wavelet leader based multifractal formalism addressed some of the caveat of previous methods, offering robust and fast estimation of the multifractal parameters.

In multifractal analysis, scaling analysis amounts to characterizing the variation of local regularity in time. The significant reduction in the spread of scaling exponents during episodes of AF suggests higher regularity of the inter-beat interval data during AF compared with other rhythms. This finding is rather counterintuitive given that AF is (by definition) an irregularly irregular rhythm. The relative regularity of AF rhythm revealed by multifractal analysis does not contradict this

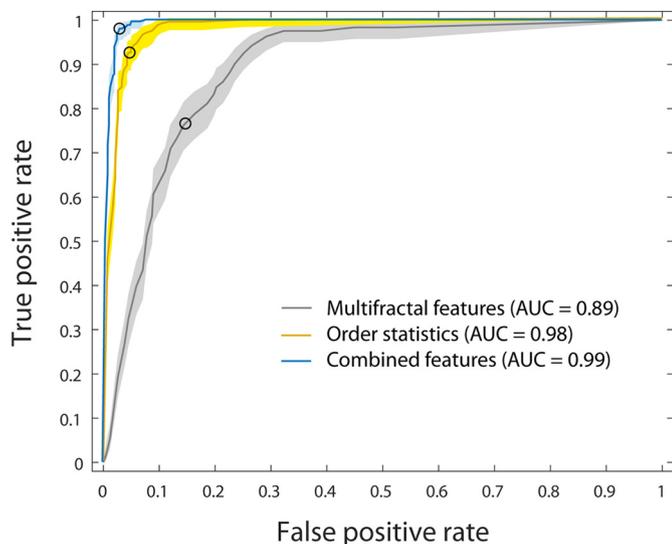


Fig. 3. ROC curves of bagged tree classification using multifractal and central moment features. AF is the positive class. AFL, J, and N are collectively the negative class. Shaded areas designate pointwise confidence bounds. Black circles indicate optimal operating points.

definition. Although smaller than in other rhythms, the width of the multifractal spectrum in AF episodes indicates an underlying multifractal (irregular) process. It does however indicate that high-order statistical properties of the inter-beat interval are more stable during periods of AF than in other periods, suggesting that some regularity seems to exist in AF (a finding supported by some of the early studies which called for the revision of the definition of AF [25]). In one study [26], reduced heart rate variability was observed in patients with AF. The authors interpreted their findings by decreased vagal tone in the heart rate regulation during AF. Although the relation of heart rate variability in AF to vagal tone has been established [27], the underpinnings of the autonomous nervous regulation in AF are still not fully understood [28].

In one recording (recording 05121, Fig. 2), many windows with AF showed relatively large multifractal spectral width values and in some windows the widths were higher than most windows of type N rhythms (other rhythms). The ECG investigation of this recording revealed that “other rhythms” contain mostly premature atrial contraction (PAC). PAC episodes can be a harbinger of AF episodes and present a rhythmicity and regularity mimicking that observed with AF, which may untimely lead to relatively similar or higher spectral widths in PAC episodes compared with AF episodes and eventually reduces the specificity of an AF classifier based on the proposed approach. It is particularly interesting to identify and investigate the multifractal properties of the rhythms encompassed under type N (other rhythms) for all recordings. Arrhythmias such as frequent multifocal atrial tachycardia, intermittent sinoatrial exit block, intermittent Mobitz, etc. that may have been labeled type N could lead to inter-beat intervals presenting similarity in multifractal properties with AF. This may indeed explain the relatively moderate specificity values obtained when training and testing the classifiers using (only) multifractal features (Fig. 3).

Other multifractal features measurable through the wavelet leader multifractal analysis are the first three log cumulants of the scaling exponents. Because theoretically these quantities are less trivial to interpret than the spread of scaling exponents, we did not include them in the statistical analysis. In principle, they characterize the linear behavior and departure from linearity in the scaling exponents.

The inclusion of these multifractal features proved to be useful in classification tasks, though accuracy of AF detection was lower than with using statistical moments. The combinations of both sets of features improved overall accuracy and achieved remarkably high sensitivity, specificity and positive predictive values when a bagged tree ensemble classifier was used. Of note, lower yet above chance classification accuracies were achieved when considering either of the other rhythms (AFL, J, and N) as the positive class. These results were not presented as they are considered out of the scope of the current study.

Compared to legacy and recently published AF detection methods which used the MIT-BIH AF database for training and/or performance testing, the proposed method shows a superior sensitivity and slightly lower specificity than 11 of 15 methods (Table 2). While two methods showed superior sensitivity and specificity [29,30] than the proposed method, they are computationally expensive as they compute Kolmogorov–Smirnov tests on histograms of inter-beat intervals. One advantage of the wavelet leader based multifractal analysis is that it has relatively low computational complexity [31]. The remarkable performance of the method proposed by Asgari et al. [32] may not be directly compared to the proposed method as it extracts features from the entire ECG waveform as opposite to inter-beat intervals.

A limitation of this study is the relatively small number of patients in the MIT-BIH atrial fibrillation database. Our motivation was to carry the analysis on a carefully annotated database to avoid annotation bias. We also sought to compare results with existing studies, which we aim to accomplish in a following study. A prospective testing of the proposed classifier on a bigger cohort of patients and longer ECG durations than in the MIT-BIH database will be required to validate the reported detection performance level.

Table 2

Comparison of the performance of the proposed approach with recently published AF detection algorithms which used the MIT-BIH atrial fibrillation database (see [32,33] for references).

Method	Sensitivity	Specificity
Asgari et al. 2015	97.0	97.1
Babaeizadeh et al. 2009 ^a	87.3	95.5
Cerucchi et al. 1997 ^a	96.1	81.5
Couceiro et al. 2008 ^a	96.6	82.7
Dash et al. 2009	94.4	95.1
Huang et al. 2011	96.1	98.1
Jiang et al. 2012	98.2	97.5
Lee et al. 2013	98.2	97.7
Linker et al. 2006 ^a	97.6	85.5
Logan et al. 2005 ^a	87.3	90.3
Moody et al. 1983 ^a	87.5	95.1
Sarkar et al. 2008	97.5	99.0
Schmidt et al. 2008 ^a	89.2	94.6
Slocum et al. 1992 ^a	62.8	77.5
Tatento et al. 2001 ^a	91.2	96.0
Proposed method	97.5	95.0

^a Results reported in Larburu et al. [33].

Another limitation of the proposed approach is the relatively large window length required for robust estimation of the multifractal features. AF episodes shorter than 5 min may not be detected which may lead to inaccurate assessment of the AF burden and limit the suitability of the proposed approach for clinical monitoring of paroxysmal AF. Whether robust assessment of the multifractal properties of inter-beat intervals using wavelet leaders on short window lengths (e.g. less than a minute) is at all possible will need to be investigated.

Conclusions

We presented a new application of a robust multifractal analysis based on wavelet leader to characterize the local regularity behavior in ECG inter-beat interval during episodes of atrial fibrillation. We have shown that significant reduction in the width of the so-called multifractal spectrum could be observed during episodes of AF compared to baseline rhythms. The combination of multifractal features derived from the wavelet leader multifractal analysis with statistical central moments (variance, and, skewness, kurtosis) led to 95% accuracy using bagged tree classifier. The proposed method may not be readily suitable for settings of continuous AF monitoring where accurate detection of AF episodes shorter than 5 min is required.

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.jelectrocard.2018.08.030>.

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